ACM contest like problems

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It is normal not to finish all the problems in 5h or less... One grade per team. No internet connexion.

1 Hamming numbers

Consider te set of Hamming number $\mathcal{H} := \left\{2^a 3^b 5^c \mid a, b, c \in \mathbb{N}\right\}$, which is a subset of $\mathbb{N}$. You are to compute the number of digits of the n^{th} element of $\mathcal{H}$ (if they are ordered according to the natural ordering on integers).

Input format

- One line containing the number of test cases;
- one line per test case, containing n ($\leqslant 200\,000$).

Sample input

```
3
1
2
3
```

Sample output

```
1
1
1
```

2 How to untrivially loose money

Carol just bought a card of a scratch game, with c cases (the price of the card is not relevant for the problem). She knows there are u cases containing a 1, t containing a 2, the other cases contain 0. They are randomly distributed, according to the uniform distribution. She can scratch any number of cases, one by one. She then wins the product of all scratched cases. She wins 1 if no case is scratched.

What is the average gain if she plays the optimum strategy ?

Input format

- One line containing the number n of test cases
- n lines containing one test case each. A test case is represented by the three integers c ($\leqslant 1200$), u ($\leqslant 1000$) and t ($\leqslant 30$), separated by spaces.

Output format One line per test case, containing the average gain rounded to two decimal digits.

Sample input

```
1
4 2 2
```

Sample output

```
4.00
```

3 Boxes

Bob has a set of boxes he'd like to insert one inside the other, in order to gain space. Each box has a marked corner, the one at pos $(0, 0, 0)$. When one box is inside another one, he wants this specific corner of the inner box to touch the specific corner of the outer one. He does not want two boxes to be side by side in another one: each box can contain at most one box, but the latter can contain another (smaller) box, which can be nonempty, and so on.

You're asked whether he can manage to have one single outer box, recursively containing all the boxes (each box being closed). The boxes have integer dimensions (in cm) and the faces have a thickness of 0.1 cm. The boxes have no specific orientation: you can turn them if needed to fit it in another box.

Input format Each test case begins with one line containing the number n of boxes, followed by n lines, one per box, containing three integers: the three dimensions of the box. The input is terminated by a line containing only a zero, which should not be processed. Every integer is at most 1000.

Output format One line per test case, containing either "`YES`" if he can have only one outer box, "`NO`" otherwise.

Sample input

```
3
4 5 6
3 4 5
1 2 3
2
1 1 5
2 2 2
0
```

Sample output

```
YES
NO
```

4 Fair paint

One wants to paint every square of an $n \times m$ rectangle either in black or white, in such a way that the black and white regions are superposable (up to rotation, translation, symmetry) and connex. Two squares are considered connex if they have a side in common, but a common corner is not sufficient.

Solutions are considered identical up to symmetry and rotation. You have to print the number of solutions.

Input format

- One line giving the number of test cases (at most 10);
- one line for each test case, consisting of $n(\leqslant 4)$ and $m(\leqslant 6)$ separated by a space.

time 30s.

Sample input

```
1
2 3
```

Sample output

```
2
```

5 Inserting operators

Consider the string "`0123456789`". You can insert one of the four symbols "`+`", "`-`", "`*`", "`/`" between any pair of consecutive digits. You can also choose not to insert anything between two digits. Parentheses are not allowed. So "`0*1+23-4*5+6+7+8-9`" is a possibility, its value is 15 $(23 - 20 + 6 + 7 + 8 - 9)$.

How many ways of inserting symbols yield the result n ?

Note: the value of "`0*1+2*3/4`" is $\frac{6}{4}$ or 1.5, but neither 1 nor 2. There will be no accumulating rounding errors (but use doubles, will you ?).

Input format Two lines, each line containing one test case, represented as the single integer n. $n < 1000$.

Output format Two lines, one for each test case.

time 2min.

Sample input

```
998
999
```

No sample output to test your code...

6 Ray-glisse

A rectangle is composed of squares, each square being either empty, a wall, or containing a capital letter. The only move permitted is to push a letter in one of the four directions. The letter then slides in that direction until it meets a wall, the border of the rectangle, or another letter (the second letter does not start moving at this point). So one move can make a letter change its position by several squares.

You are to compute the minimum number of moves to go from one given position to another given position. You can assume the end position can be reached.

Input format A test case is represented by two integers n and m ($\leqslant 7$) on one line separated by a space, then the start position, then the end position. Those two integers n and m are the dimension of the rectangle. A position is represented by n lines of m characters, each character being either "`#`" for a wall, "`.`" for an empty square, or a capital letter. There are at most 4 letters in the rectangle. A position is always followed by a blank line.

You can assume that the "`#`" are the same in the start and end positions, and that each letter appears the same number of times in the start and end positions. The input is terminated by a line containing two "0" (for n and m), this line should not be processed.

time 30s.

Sample input

```
1 3
B..

..B

3 3
#..
...
.AA

#..
.A.
A..

0 0
```

Sample output

```
1
4
```